MATH 54 - HINTS TO HOMEWORK 7

PEYAM TABRIZIAN

Here are a couple of hints to Homework 7! Enjoy :)

NOTE: In case you're stuck with the more standard differential equations questions (hom equations, undetermined coeffs, variation of parameters, etc.), make sure to look at my 'Second-Order Differential equations' and 'Higher-Order Differential Equations'-Handouts. They should help you solve all the problems!

SECTION 4.2: HOMOGENEOUS LINEAR EQUATIONS: THE GENERAL SOLUTION

4.2.26. This problem looks scary, but it's not that scary! In each question, try to solve for c_1 and c_2. In (a), you'll be able to do that, in (b), there will be no solutions, and in (c) there will be infinitely many solutions!

4.2.27, 4.2.28. Here's a useful trick which I showed in lecture. Use the Wronskian:

$$W(t) = det \begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix}$$

You **DON'T** have to calculate this determinant explicitly. Just pick a point t_0 between 0 and 1 where $W(t_0) \neq 0$ (say $\frac{1}{2}$ or $\frac{\pi}{4}$) If $W(t_0) \neq 0$, then your solutions are linearly independent.

Also, for 4.2.27, first try to simplify for functions! For example, if $ae^{-t}\cos(2t) + be^{-t}\sin(2t) = 0$, then you can cancel out the e^{-t} which gives you $a\cos(2t) + b\sin(2t) = 0$. The point is that you only have to show that $\cos(2t)$ and $\sin(2t)$ are linearly independent! This should simplify the calculation of the Wronskian!

SECTION 4.3: AUXILIARY EQUATIONS WITH COMPLEX ROOTS

The problems should be pretty straightforward :) Ignore $4.3.33(c)$!

SECTION 4.4: THE METHOD OF UNDETERMINED COEFFICIENTS

Note: Don't worry about that weird trick about multiplying your solution by t or not. I will not ask you about this weird trick on this exam!

Date: Thursday, July 12th, 2012.

4.4.9. Guess $y_p(t) = A$

4.4.11. $y_p(t) = Ae^{2t}$

4.4.13. $y_p(t) = A\cos(3t) + B\sin(3t)$

4.4.15. $y_p(x) = (Ax + B)e^x$

4.4.28. $y_p(t) = (At^4 + Bt^3 + Ct^2 + Dt + E)e^t$ (you always guess the most complicated solution possible)

4.4.30. $y_p(t) = Ae^t\cos(t) + Be^t\sin(t)$ (remember that whatever you do with $\cos$, you always have to repeat it with $\sin$)

SECTION 4.5: THE SUPERPOSITION PRINCIPLE

4.5.17. For the particular solution, guess $y_p(t) = At + B$

4.5.30. First find the general solution of $y'' + 2y' + y = 0$, then find a particular solution to $y'' + 2y' + y = t^2 + 1$ (for this, guess $y_p(t) = At^2 + Bt + C$), and then find a particular solution to $y'' + 2y' + y = -e^t$ (for this, guess $y_p(t) = Ae^t$), and add all 3 solutions together. Finally, solve for the constants using $y(0) = 0, y'(0) = 2$

4.5.32. $y_p(t) = Ae^{2t} + (Bt + C)e^{2t} + (Dt^2 + Et + F)e^{2t}$

SECTION 4.6: VARIATION OF PARAMETERS

The easiest way to do the problems in this section is to look at my differential equations handout!

The formula is:

Let $y_1(t)$ and $y_2(t)$ be the solutions to the homogeneous equation, and suppose $y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$. Let:

$$\widetilde{W}(t) = \begin{bmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{bmatrix}$$

And solve:

$$\widetilde{W}(t)\begin{bmatrix} v_1'(t) \\ v_2'(t) \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

where $f(t)$ is the inhomogeneous term.

4.6.12. Don't use variation of parameters to find the complete particular solution! First find the general solution to $y'' + y = 0$, then use var. of par. to find a particular solution of $y'' + y = \tan(t)$, and then use *undetermined coefficients* to find a particular solution of $y'' + y = e^{3t}$ and use undetermined coefficients again to find a particular solution of $y'' + y = -1$, and add the 4 solutions you found together!

SECTION 6.1: BASIC THEORY OF LINEAR DIFFERENTIAL EQUATIONS

6.1.1, 6.1.5. First of all, make sure that the coefficient of y''' is equal to 1. Then look at the domain of each term, including the inhomogeneous term (more precisely, the part of the domain which contains the initial condition -2 resp. $\frac{1}{2}$). Then the answer is just the intersection of the domains you found! For a sample-problem, look at the Higher-Order Differential Equations Handouts!

6.1.7. Use the Wronskian:

$$W(t) = det \begin{bmatrix} y_1(t) & y_2(t) & y_3(t) \\ y_1'(t) & y_2'(t) & y_3'(t) \\ y_1''(t) & y_2''(t) & y_3''(t) \end{bmatrix}$$

You **DON'T** have to calculate this determinant explicitly. Just pick a point t_0 where $W(t_0) \neq 0$ (in 6.1.7, take $x = 0$) If $W(t_0) \neq 0$, then your solutions are linearly independent.

6.1.9. $\cos^2(x) + \sin^2(x) = 1$, so linearly dependent . The point is that you generally use the Wronskian to show linear independence. Linear dependence is in general easier to check!

6.1.11. Use the Wronskian with $x = 1$ (here you have to do a Wronskian with 4 functions, but that's similar to the 3 function-case!

6.1.15. You have to check 2 things: First show that each function actually solves the differential equation, then use the Wronskian with $x = 0$ to show that the functions are linearly independent. Then the general solution is $y(x) = Ae^{3x} + Be^{-x} + Ce^{-4x}$.

6.1.34. You do **NOT** have to evaluate the determinant!!! If you (in theory) expand the determinant along the last column, you should (in theory) get a third order differential equation. The coefficient of y''' is the Wronskian of f_1, f_2, f_3, which is nonzero (hence we really get a third-order ODE). Notice that if you plug $y = f_1$ into the Wronskian, then the first and the fourth columns are identical, hence linearly dependent, which shows that the determinant is in fact 0, so f_1 indeed solved the differential equation. Similarly for f_2 and f_3. Hence $\{f_1, f_2, f_3\}$ is indeed a fundamental solution set!

6.1.35. Here you have to evaluate the determinant in 34, with $f_1 = x, f_2 = \sin(x), f_3 = \cos(x)$, but in this case the determinant should be easier to evaluate because the first column has two zeros!

SECTION 6.2: HOMOGENEOUS LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS

6.2.1, 6.2.3, 6.2.9. The following fact might be useful:

Rational roots theorem: If a polynomial p has a zero of the form $r = \frac{a}{b}$, then a divides the constant term of p and b divides the leading coefficient of p.

This helps you 'guess' a zero of p. Then use long division to factor out p.

Also check out the 'Higher-order differential equations'-Handout for a sample problem!

6.2.15, 6.2.17. The reason this is written out in such a weird way is because the auxiliary polynomial is easy to figure out! For example, in 6.2.15, the auxiliary polynomial is

$(r-1)^2(r+3)(r^2+2r+5)^2$.

6.2.19, 6.2.21. See hints to 6.2.1

6.2.25. Suppose:

$$a_0e^{rx} + a_1xe^{rx} + \cdots + a_{m-1}x^{m-1}e^{rx} = 0$$

Now cancel out the e^{rx}, and you get:

$$a_0 + a_1x + \cdots + a_{m-1}x^{m-1} = 0$$

But $1, x, x^2 \cdots, x^{m-1}$ are linearly independent, so $a_0 = a_1 = \cdots a_{m-1} = 0$, which is what we wanted!

6.2.26. Follow the hints, I think it's a really neat problem :)